FOUR ADVANCES IN HANDLING UNCERTAINTIES IN SPATIAL DATA AND ANALYSIS

Wenzhong Shi

Advanced Research Centre for Spatial Information Technology
Department of Land Surveying and Geo-Informatics
The Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong
Fax: +852 2330 2994; E-mail: lswzshi@polyu.edu.hk

KEY WORDS: uncertainty, quality control, spatial data, spatial analysis, spatial model

ABSTRACT:

Data quality and uncertainty modeling for spatial data and spatial analyses is regarded as one of the disciplines of geographic information science together with space and time in geography, as well as spatial analysis. In the past two decades, a lot of research efforts have been devoted to uncertainty modeling for spatial data and analyses, and this paper presents our work in this research area. In particular, four progresses in the area are given: (a) from *determine*- to *uncertainty*-based representation of geographic objects in GIS; (b) from uncertainty modeling for *static* data to *dynamic* spatial analyses; (c) from modeling uncertainty for spatial *data* to *models*; and (d) from error *descriptions* to quality *control* for spatial data.

1. INTRODUCTION

Three areas are defined as the major principles of geographic information science: (a) space and time in GIS, (b) data quality, and (c) spatial analysis (Longley *et al*., 1999). In fact, uncertainty modeling covers these three areas, besides the most obvious one – data quality. Space and time modeling in GIS should not only for determined spatial objects, but also cover uncertain spatial objects. In fact, these are two fundamental types of spatial objects in the real world and which can be modeled in GIS, although most of the existing GIS mainly handle determined spatial objects. Spatial analyses in GIS are the major functions for the spatial related decision making. Such spatial analyses are not error free. Uncertainty in the source data and limitations of the spatial analysis models may propagate and further introduce uncertainties in the spatial analysis processes.

With these considerations, we have devoted ourselves to uncertainty modeling for spatial data and spatial analyses in the past few years and have formed a framework (Shi, 2005). A number of research progresses have been made in the uncertainty modeling through the studies. This paper intends to introduce four major theoretical breakthroughs in uncertainty modeling, namely (a) advances in spatial objects representation -- from *determine*- to *uncertainty*-based representation of geographic objects in GIS; (b) from uncertainty modeling for *static* spatial data to *dynamic* spatial analyses; (c) from uncertainty modeling for spatial *data* to spatial *models*; and (d) from error *description* of spatial data to spatial data quality *control*. In fact, there are a large number of research achievements made by other international peers in these four areas, but these will not be introduced in this paper due to the limitation of the paper length. In this paper, I will mainly summarize the progresses in these four areas made by us.

The rest of this paper will detail these new progresses in uncertainty modeling for spatial data and spatial analyses.

2. FROM DETERMINE- TO UNCERTAINTY-BASED REPRESENTATION OF OBJECTS IN SPACE

One of the fundamental issues in geographic information science is the representation of space and time. Euclidean space is regarded as one of the applicable spaces for representing geographic entities and phenomena, where the basic elements are points, lines, areas and volumes of the space.

In fact, geographic phenomena can be classified as two classes: determined and uncertain. According to the nature of uncertainties in geographic lines, Shi (1994) classified the geographic lines into two types: Type I line and Type II line. The fundamental difference between these two types of lines is that there are "real points" in the real world which construct a Type I line, while there is no real world point for a Type II line. Thus we have to determine or interpret these points for a Type II line by ourselves. An example of a Type I line is a building boundary, while a boundary between a forest and grassland is an example of a Type II line. In principle, we should use a determine-based representation for a Type I line; on the other hand, we should apply an uncertainty-based representation for a Type II line. Unfortunately, we only have determine-based representation model for most of the current GIS. Therefore, there is room to further develop uncertainty-based spatial representations for geographic objects.

In our development, the uncertainty-based spatial representations of the geographic entities and phenomena cover points, line segments/ straight lines, curve lines and polygons.

2.1 Uncertainty-based Representation of Points

Instead of a determined point represented by its coordinates (x, y), a point with uncertainty can be represented by its confidence region, error ellipse, or error distribution of the point.

(a) Confidence Region Model for a Point

The confidence region of a point gives a region around the measured point, which contains the true location of the point with the probability that is larger than a predefined confidence level.

(b) Error Ellipses and Distribution

If the errors in the coordinates of a point are assumed to be normally distributed, their two-dimensional probability density function can be formed, and the error of the point coordinates is indicated by the standard ellipses. The probability that the point is inside the error ellipse is estimated by the volume of the two-dimensional error curved surface over the error ellipse. This probability increases with the size of the error ellipse.

2.2 Uncertainty-based Representation of Lines

(a) Confidence Region Model for a Two-dimensional Line Segment

A confidence region for a two-dimensional line segment is defined as a region containing the true location of the line segment with a predefined confidence level (Shi, 1994). This confidence region J_2 is the union of the confidence regions J_{2r} for all points Q_{2r} on the line segment for $r \in [0, 1]$, so that the true locations $\varnothing_{2r}$ of all points on the line segment are contained within J_2 with the probability larger than a predefined confidence level:

$$P(\varnothing_{2r} \in J_{2r} \text{ for all } r \in [0, 1]\,) > \alpha \tag{1}$$

The confidence region J_{2r} is a set of points $(x_1, x_2)^T$ with x_1 and x_2 satisfying

$$X_{1r} - c_{21} \leq x_1 \leq X_{1r} + c_{21} \tag{2}$$
$$X_{2r} - c_{22} \leq x_2 \leq X_{2r} + c_{22} \tag{3}$$

where
$$c_{21} = [k((1\text{-}r)^2 + r^2)]^{1/2}\sigma_1$$
$$c_{22} = [k((1\text{-}r)^2 + r^2)]^{1/2}\sigma_2$$
$$k = \chi^2_{2;(1+\alpha)/2}$$

Figure 1 shows the confidence region J_2 for a measured location $Q_{21}Q_{22}$ of a line segment with a predefined confidence level α = 0.97. Points Q_{21} and Q_{22} are measured locations of the endpoints of the line segment, while points $\varnothing_{21}$ and $\varnothing_{22}$ are their corresponding true locations.

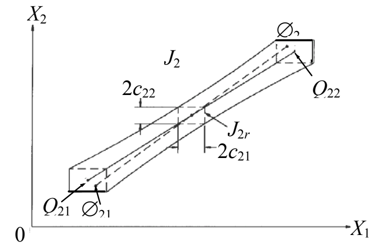

Figure 1. The confidence regions of a two-dimensional line segment (after Shi, 1994)

Based on the confidence region model for a line segment, the confidence region for a two-dimensional polyline, or a polygon, were also built. Furthermore, the confidence region model was also extended to a generic one for N-dimensional lines (Shi, 1998).

(b) Probability Distribution Model for a Line Segment

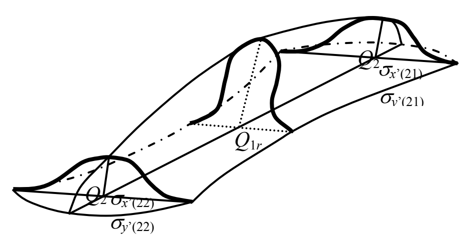

Figure 2. The probability density function of line segment $Q_{21}Q_{22}$ (after Shi 1994)

The probability analysis can be applied to depict the error distribution of that line. Based on the probability analysis, positional error on any straight line is modeled from a joint probability function of those points on the straight line. This joint probability function gives the probability distribution for the line over a particular region. The probability of the line falling inside the corresponding region is defined as the volume of the line's error curved surface formed by integrating the error curved surfaces of individual points on the line.

(c) The G-band Error Model

The G-band error model provides the quantities and characteristics of the error of the points on the line segment. Referring to Figure 2, if the surface for the probability density function of line segment Q21Q22 is cut by the plane that is parallel to the x-O-y plane and projected to the x-O-y plane, a class of concentric bands will be obtained. The concentric band which is cut by the plane passing through the error ellipses of the endpoints of the line segment is defined as the generic error band, known as the G-band (Shi and Liu, 2000). The model describes the error of an arbitrary point on the line segment, given the error of the two end points are either correlated or independent. The model also gives the analytical relationships between the band shape and size with the error at the two endpoints and relationships between them.

2.3 Uncertainty-based Representation of Curves

The positional uncertainty of a curve feature is described by two error indicators: (a) the ε_σ -- to measure the error in any point on the curve in the direction of the curve normal, and (b) the ε_m -- to describe the maximum error at the point to the curve. Both of them can assess positional error on a curve feature. The two error indicators are applied to describe uncertainty of both a regular curve (such as a circular curve) and an irregular curve (such as a third-order spline curve) (Shi, Tong, and Liu, 2000).

2.4 Uncertainty-based Representation of Polygons

Positional error for a polygon in GIS is caused by positional error in its boundary, mainly from the component vertices of the polygon. Two approaches on positional error modeling for a polygon are given: the first approach is based on error of the component vertices of the polygon, and the second approach is based on error of the component line segments through their error bands (Shi, 2005).

(a) Error Modeling Based on Its Component Vertices

A polygon is composed of its component vertices. Error of a polygon is thus determined by error of its vertices. Error of the polygon is quantified according to the error of its vertices based on error propagation law in statistics. A polygon is also described by its parameters, such as its area, perimeter, gravity point, etc. Therefore, uncertainty of a polygon is described by these parameters.

(b) Error Modeling Based on Its Component Line Segment

In fact, the area of a polygon is also a function of length of the component line segments of the polygon. This implies that positional error for the area of the polygon is subject to positional error in these line segments. The relationship of the positional error in the line segment and the error in the area of the polygon are quantified, and uncertainty of the polygon can thus be estimated based on the error of the component line segments.

3. FROM MODELING UNCERTAINTY IN STATIC DATA TO DYNAMIC SPATIAL ANALYSES

Here, spatial analysis refers to the GIS spatial analyses, such as overlay analysis, buffer analysis, line simplification, projection etc. In fact, each of these spatial analyses is a transformation based on one or more original spatial data set(s). Uncertainty inherited in the original data sources will be further propagated or even amplified through such a spatial analysis. Sometimes, new uncertainties will be generated through such a spatial analysis. Therefore, besides modeling uncertainty in static spatial data, a step further for our research in this area is to model uncertainty in spatial analysis.

3.1 Modeling Uncertainty in Overlay Spatial Analysis

Modeling uncertainties in overlay spatial analysis provides an estimation of the uncertainty propagated through the analysis. Two methods are proposed to estimate the propagation of errors in vector-based overlay spatial analysis: an analytical error model which is derived by the error propagation law, and a simulation error model (Shi, Cheung, and Tong, 2004). For each of these two error models, it is proposed that the positional error in the original or derived polygons be assessed by three measures of error: (a) the variance-covariance matrices of the polygon vertices, (b) the radial error interval for all vertices of the original or derived polygons, and (c) the variance of the perimeter and that of the area of the original or derived polygons. The variance-covariance matrix of the vertices of an original or derived polygon is a relatively comprehensive description of error. The radial positional error interval is proposed and it is more practical for describing the error at the vertices of a polygon.

The experimental study results demonstrated that the number of vertices and the error at the vertices of a polygon are two major factors that affect the accuracy of parameters of the polygon such as perimeter and area. Increasing (or decreasing) both the number and the error of the vertices of a polygon will similarly influence the error of these polygon parameters.

3.2 Modeling Uncertainty in Buffer Spatial Analysis

Modeling uncertainty in buffer spatial analysis provides a solution of quantifying uncertainty propagation through a buffer spatial analysis. A method of modeling the propagation of errors in a buffer spatial analysis for a vector-based GIS has been developed (Shi, Cheung, and Zhu, 2003). Here, the buffer analysis error is defined as the difference between the expected and measured locations of a buffer. The buffer can be, for example, a point, line-segment, linear, or area features. The sources of errors in a buffer analysis include errors of the composed nodes of points or linear features, as well as errors of buffer width. These errors are characterized by their probability density function.

Four indicators of error and their corresponding mathematical models, in multiple integrals, have been proposed for describing the propagated errors in a buffer spatial analysis. These include the error of commission, error of omission, discrepant area, and normalized discrepant area. Both the error of commission and the error of omission are suitable for describing a situation where the expected and measured buffers overlap. The discrepant area indicator has been defined, taking into consideration the possibility that the expected and measured buffers might not overlap with each other. This error indicator in fact provides a more generic solution for the cases where the measured and expected buffers either overlap with each other or do not. Furthermore, the normalized discrepant area has been proposed, considering the mathematical rigors of the definition – to meet the conditions of the definition of distance in algebra.

3.3 Modeling Uncertainty in Line Simplification

Modeling uncertainty in line simplification provides a solution on uncertainty estimation for a line simplification or generalization process. The error sources of a line simplification are: (a) the uncertainty in an initial line, and (b) the uncertainty due to the deviation between the initial and simplified lines. We classified the uncertainties in a line simplification process as a combination of the propagated uncertainty, the modeling uncertainty, and the overall processing uncertainty (Cheung and Shi, 2004). The propagated uncertainty is used to identify the uncertainty effect of the initial line, and the modeling uncertainty represents the uncertainty arising from the line simplification process. The overall uncertainty in the simplified line is modeled by the overall processing uncertainty that integrates both the propagated and the modeling uncertainties in the line simplification process.

Three uncertainty indices and corresponding mathematical solutions were proposed for each type of uncertainty by measuring its mean, median, and maximum values. For the propagation uncertainty, we proposed the mean discrepancy, the median discrepancy, and the maximum discrepancy; for the modeling uncertainty, we proposed the mean distortion, the median distortion, and the maximum distortion; and for the overall processing uncertainty, we proposed the mean deviation, the median deviation, and the maximum deviation.

The distributions of all of the types of uncertainty are positively skewed. The mean uncertainty index provides the general value of the type of uncertainty. The relation between the mean uncertainty index for the overall processing uncertainty and the threshold distance in the DP line simplification was studied. It was found that the mean uncertainty index is a monotonic increasing function of the threshold distance. Also, this function is used to determine the threshold distance such that an uncertain simplified line is close to the "true" initial line to a predefined acceptable level of accuracy.

4. FROM MODELING UNCERTAINTY FOR SPATIAL DATA TO SPATIAL MODELS

In stead of describing real world objects by data, real world objects can also be described by spatial models. For example, digital terrains can be represented by digital elevation models. The models can be either a regular one, such as regular tessellation like a square, or an irregular one, such as Triangulated Irregular Network (TIN). Therefore, it is a natural evolution of uncertainty modeling to go: from spatial data to spatial models.

The difference between what in the real world and what represented by the models in GIS, is referred to model uncertainty. Here in this paper, uncertainty modeling of DEM model is taken as an example of model uncertainty estimation in GIS. Two achievements have been made along the lines of model accuracy estimation for DEM: (a) a formula to estimate the average model accuracy of a TIN, and (b) accuracy estimation of bicubic interpolation model.

4.1 Model Accuracy Estimation for TIN

TIN is a widely used model for the representation of digital terrain by irregular triangles due to its higher accuracy of terrain representation and with the considerations of feature lines in the model. However, this does not mean there is no error in a TIN model. The mean error of the DEM is identified and is estimated by the following mathematical formula (Zhu, Shi *et al.*, 2005):

$$\sigma_H^2 = \frac{1}{2}\sigma_{node}^2 . \tag{4}$$

As an estimator of overall accuracy, the mean error of TIN is invariant to both the shape and spatial locations of triangles. From the formula, it can be determined that the mean elevation accuracy in a triangle is dependent on the error σ_{node}^2 of the original data but is independent of the shape and size of the triangle.

4.2 Model Accuracy Estimation for DEM from High-order Interpolation

Here, model accuracy estimation for regular grid DEM is given by taking the bicubic interpolation model as an example.

In fact, error of an interpolated DEM related to (a) the model error of the interpolation algorithm and (b) the error of the original source data – coordinates of the nodes in the network. The formulae for estimating the propagated mean elevation error of both the biquadtric and bicubic interpolation models are derived based on rigorous mathematical approval (Shi *et al.*, 2005). The results indicated that the propagated mean elevation error from a biquadratic interpolation is identical with that of a bicubic interpolation. Furthermore, they are the same as that of a bilinear interpolation model. The propagation errors, in terms of the mean elevation error of a DEM surface (in a root mean square error), can be expressed by the following formulae for both biquadratic and bicubic interpolations:

$$\sigma_{DEM\ Surface}^2 = \frac{4}{9}\sigma_{node}^2 . \tag{5}$$

According to the result of the estimation of model error by Kidner (2003), the higher-order interpolation generates up to 20% fewer model errors compared with those of the bilinear interpolation. Therefore, it can be concluded that both the biquadratic interpolation and bicubic interpolation are more accurate than the bilinear interpolation in terms of their total errors – including both the propagation error and the model error of a generated DEM surface.

5. FROM UNCERTAINTY DESCRIPTION TO SPATIAL DATA QUALITY CONTROL

Most of the early studies mainly focused on describing uncertainties, including describing uncertainty in spatial data, describing uncertainty in a spatial analysis, or describing uncertainty in a spatial model. From an uncertainty handling point of view, description is a necessary first step; a further step is to control or even reduce the uncertainties in the spatial data, analyses or models, if we are able to do so. Therefore, another progress made in handling uncertainties in spatial data and spatial analysis is from uncertainty description to quality control of spatial data and analyses. In this regard, the quality control for the following three data and models are given in this paper:

- Quality control for object-based spatial data – to control overall geometric quality of vector spatial data by the least square adjustment methods;
- Quality control for field-based spatial data – to geometrically rectify high resolution satellite imageries by point- and line-based transformation models; and
- Quality control for digital elevation models – to improve DEM accuracy by newly proposed hybrid interpolation model.

5.1 Quality Control for Object-based Spatial Data

This section briefs the principles of quality control for object-based spatial data, taking the vector cadastral data as an example (Tong, Shi and Liu, 2005). The statistical method for the spatial data quality control is the least square adjustment. The least squares method is adopted to solve inconsistencies between areas of digitized and registered land parcels, which has been proposed for adjusting the boundaries of the land parcels.

In this regard, the land parcel is the target to be adjusted in the data quality control process. A systematic strategy has been proposed for the cadastral data quality control by the least square adjustment, including: (a) to let the registered area in a land parcel as a true value and a digitized point with coordinates as the observations for the establishment of the adjustment conditional equations; (b) to let both the registered area and the digitized point with coordinates as the observations, the weights of these two types of observation are estimated based on the variance components; and (c) to add the scale factor of the land parcel in the adjustment model and to estimate the weight of the observations based on the variance component from conditional adjustment with parameters. The adjustment for the land parcels is then realized.

For example, in method (a), the size of the registered area of a land parcel is taken as its true value and we adjust the geometric position of the boundaries of the digitized parcel. An area adjustment model is derived by incorporating the following two categories of constraints: a) attribute constraints: the size of the true area of the parcel, and b) geometric constraints: such as straight lines, right angles, and certain distances. Second, the method is then used to adjust the areas of the parcels.

The quality control method (b) is further developed due to the considerations that both digitized coordinates of a land parcel and the registered area of the land may contain errors, although the quantity may be different. Now, the problem becomes how to solve the inconsistencies between the digitized and registered areas of land parcels under the condition that both of them are treated as observations with errors. Here, the key issue is to determine the weights of these two types of observations. The least squares adjustment, based on the Helmert method, is proposed for estimating the weights between these two types of observations. The insistency between the registered area and digitized area of the parcel is then adjusted through the least squares adjustment.

It is demonstrated, via several applications of the solution, that the proposed approaches are able to solve the problem of data inconsistency between the digitized area value and registered area value of the land parcels. This method has, in fact, solved one of the most critical problems in a vector GIS – data inconsistency in the databases.

5.2 Quality Control for Field-based Spatial Data

Satellite imagery is a kind of field-based spatial data. Due to the terrain effects, projection of imaging, and image scanning, as well as positioning error of the satellites during image capture, the geometry of an originally obtained satellite image contains positional errors. To control the quality and improve the positional accuracy of the obtained images, a geometric rectification process needs to be applied to the images. Instead of the traditional, point-based approach, line-based transformation model is proposed as a further development (Shi and Shaker, 2006).

(a) Point-based Geometric Rectification

In the point-based geometric rectification methods for improving the quality of the satellite image, the ground control points (GCPs) are used to build transformation models – a relationship between satellite imaging coordinate system and the ground coordinate system. Here, GCPs are obtained either through ground surveying by GPS, or by using those control points from maps. The transformation models can be, for example, polynomials, affine transformation model, or other mathematical models. These models with known parameters are then used to rectify the original satellite images – to control their quality by improving the positional accuracy.

A transformation model (non-rigorous models) from a 2D satellite image coordinate system to ground 3D object space is studied (Shi and Shaker, 2003; Shaker, Shi, and Barakat, 2005), and the transformation model is applied to the geo-positioning of IKONOS imagery. In particular, the following two factors that affect the data quality control are investigated: a) change in terrain heights, and b) the number of GCPs with 3D ground coordinates.

It is found that for the 2D transformation models, the rectification accuracy can be improved if the ground point coordinates are corrected by projecting them into a compensation plane before the 2D IKONOS imagery is geometrically rectified. It is also found that the second order polynomial model presented the best positional accuracy results with a modest number of GCPs required.

The results of the affine model showed that a one-meter 3D ground point determination accuracy is achievable for stereo images without any need to obtain further information about the satellite sensor model and ephemeris data. Furthermore, increasing the number of GCPs significantly improves the accuracy of the results when the affine model is applied to an area with different types of terrain, such as in the Hong Kong study area with its flat, hilly, and mountainous types of terrain.

(b) Line-based Transformation Model

In many cases, the precise GCPs are not available, but lines are available from maps, for example. The line-based rectification method, the Line-Based Transformation Model (LBTM) for image-to-image registration, and image to 3D ground space transformation, has been proposed (Shi and Shaker, 2006). This new model builds the relationship between image coordinate systems and ground coordinates systems using unit vector components of the line segments of line features. The LBTM is in two forms: a) 2D affine LBTM, and b) 2D conformal LBTM.

Several experiments showed that both forms of the LBTM are applicable for the image-to-image registration of high-resolution satellite imagery. In particular, the results revealed that an accuracy of better than two pixels can be achieved using a moderate number of GCLs and the LBTM.

5.3 Quality control for DEM data

Quality of a DEM is determined by the model used to represent the terrain. Therefore, the quality of a DEM can be controlled or improved by a better designed interpolation model. A hybrid interpolation method by integrating conventionally used linear and non-linear interpolation models has been proposed to improve interpolation accuracy (Shi and Tian, 2006) with the following mathematical expression:

$$I = \rho A + (1 - \rho) B, \ (0 \le \rho \le 1) \tag{6}$$

The hybrid parameter (ρ) adjusts both the linear and non-linear components in the hybrid model. The value of the hybrid parameter is related to the complexity level of a terrain. This model was proposed based on an understanding of the both the low- and high-frequency components contained in the surface of real world terrains.

The experiments showed that the hybrid model can generate more accurate DEM than that from the bilinear and bi-cubic methods in terms of RMSE. The proposed hybrid interpolation method provides an alternative solution to the existing DEM interpolation methods, combining the advantages of both linear and nonlinear interpolation models.

6. CONCLUDING REMARKS

Within the framework of handling uncertainties in spatial data and analyses, this paper presents four progresses we made so far: (a) from determine- to uncertainty-based spatial representation geographic entities in space; (b) from uncertainty modeling for static data to dynamic analyses; (c) from uncertainty modeling for simple data to comprehensive models; and (d) from passive uncertainty description to active quality control for spatial data.

In the future, the following research issues are suggested: (a) the effect of uncertainties in spatial data to the result of decision making; (b) application- or service-oriented spatial data quality models and presentation; (c) spatial data quality control; (d) implementing of the theoretical development in commercial GIS and other software; and (e) to introduce the newly developed methods for assessing spatial data quality in national and international standards.

ACKNOWLEDGEMENTS

The work described in this paper was supported by grants from the National Natural Science Foundation, China (Project No. 406 29 001) and The Hong Kong Polytechnic University (G-YF24).

REFERENCES

Cheung, C.K, W.Z. Shi, 2004. Estimation of the positional uncertainty in line simplification in GIS. *The Cartography Journal*, Vol. 41, No.1, pp: 37-45.

Kidner, D.B., 2003. Higher-order interpolation of regular grid digital elevation models. *International Journal of Remote Sensing*, 24, pp: 2981-2987.

Longley, P.A., M.F. Goodchild, D.J. Maguire and D.W. Rhind, 1999. *Geographical Information Systems, Principles and Technical Issues (Volume 1)*. John Wiley & Sons, Inc., New York, Chichester, Weinheim, Brishane, Sigapore and Toronto.

Shaker, A., W.Z. Shi and H. Barakat, 2005. Assessment of the rectification accuracy of Ikonos imagery based on two-dimensional models. *International Journal of Remote Sensing*, Vol. 26, No.4, pp: 719-731.

Shi, W.Z., 1994. *Modeling Positional and Thematic Uncertainties in Integration of Remote Sensing and Geographic Information Systems*. ITC: Publication No.22, Enschede, ISBN 90 6164 099 7, 147 pages.

Shi, W.Z., 1998. A generic statistical approach for modeling error of geometric features in GIS. *International Journal of Geographic Information Science*, Vol. 12, pp: 131-143.

Shi, W.Z., 2005. *Principle of modeling uncertainties in spatial data and analysis*. Science Press, Beijing, ISBN: 7-03-015602-1, 408 pages.

Shi, W.Z., A. Shaker, 2003. Analysis of terrain evaluation effects on IKONOS imagery rectification accuracy by using non-rigorous models. *Photogrammetric Engineering and Remote Sensing*, Vol. 69, Number 12, pp: 1359-1366.

Shi, W.Z., A. Shaker, 2006. The Line-Based Transformation Model (LBTM) for image to image registration of high-resolution satellite image data. *International Journal of Remote Sensing*, Vol. 27, No. 14, pp: 3001-3012.

Shi, W.Z., C.K. Cheung and C.Q. Zhu, 2003. Modeling error propagation in vector-based buffer analysis. *International Journal of Geographic Information Science* Vol. 17, No. 3, pp: 251-271.

Shi, W.Z., C.K. Cheung and X.H. Tong, 2004. Modeling error propagation in vector-based overlay spatial analysis. *ISPRS Journal of Photogrammetry and Remote Sensing*, Vol. 59, Issues 1-2, pp: 47-59.

Shi, W.Z. and Liu, W.B., 2000. A stochastic process-based model for the positional error of line segments in GIS. *International Journal of Geographic Information Science*, Vol. 14, pp: 51-66.

Shi, W.Z., Q.Q. Li, C.Q. Zhu, 2005. Estimating the propagation error of DEM from higher-order interpolation algorithms. *International Journal of Remote Sensing*. Vol. 26, No. 4, pp: 3069-3084.

Shi, W.Z., Tong, X.H. and Liu, D.J., 2000. An approach for modeling error of generic curve features in GIS. *Acta Geodaetica et Cartographica Sinica*, VOl. 29, pp: 52-58 (in Chinese).

Shi, W.Z., Y. Tian, 2006. A hybrid interpolation method for the refinement of regular grid digital elevation model. I*nternational Journal of Geographical Information Science*, Vol. 20, No. 1. pp: 53-67.

Tong, X.H., Shi, W.Z. and Liu, D.J., 2005. A least squares-based method for adjusting the boundaries of area objects. *Photogrammetry Engineering and Remote Sensing*, Vol. 71 No.2, pp: 189-195.

Zhu, C.Q., W.Z. Shi, Q.Q. Li, G.X. Wang, T.C.K. Cheung, E.F. Dai and G.Y.K. Shea, 2005. Estimation of average DEM accuracy under linear interpolation considering random error at the nodes of TIN model., *International Journal of Remote Sensing*, Vol. 26, No. 24, pp: 5509-5523.